

Operator Analysis for Proton Decay in SUSY SO(10) GUT Models

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Non-renormalizable operators both account for the failure of down quark and charged lepton Yukawa couplings to unify and reduce the proton decay rate via dimension-five operators in minimal SUSY SU(5) GUT. We extend the analysis to SUSY SO(10) GUT models.

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The standard model (SM) is a very successful but only effective theory, which has to be extended at higher energies. Grand Unified Theories provide a beautiful framework for a more fundamental theory because the additional symmetries of the underlying group G_{GUT} restrict some of the arbitrary features, in particular via relations between Yukawa couplings. Unfortunately, these relations are troublesome in simple GUT models like minimal SU(5) [1]: The equality $m_d = m_e$ is correct for the third but fails for the first and second generation. It was, however, noticed early that since the GUT scale M_{GUT} is close to the Planck scale M_{P} , it is natural to expect corrections to the fermion mass matrices, which do not respect this equality [2]. Moreover, these corrections are important for supersymmetric GUT models because they can significantly reduce the proton decay rate via dimension-five operators. For minimal supersymmetric SU(5) [3], they are sufficient to make it consistent with the present experimental bound [4, 5].

Since these additional contributions are important, it is reasonable to study them in more detail. In particular, we would like to know, whether they are totally arbitrary from the GUT model's point of view, or whether there is any mechanism which would naturally lead to the required relations among Yukawa couplings. We can think of two possibilities, the first of which is to start with some ad-hoc textures as a result of an unknown additional symmetry [6]; the second is to extend the analysis to another group, in order to obtain additional symmetry restrictions.

For the latter approach, SO(10) [7] is a natural choice since it unifies the matter fields and involves massive neutrinos. It is broken to the SM either via the Pati-Salam group [8] or SU(5) \times U(1). Higher-dimensional operators have been studied in the former case before (see e.g. Refs. [9, 10]); the purpose of this letter is to study the non-renormalizable operators for the second case. Here, these operators have been studied in parts only, see e.g. Ref. [11, 12].

At the beginning, we review the higher-dimensional operators in supersymmetric SU(5). The minimal model

contains three generations of chiral matter multiplets, $10 = (Q, u^c, e^c)$, $5^* = (d^c, L)$, and as Higgs fields, an adjoint multiplet $\Sigma(24)$ and a pair of quintets, $H(5)$ and $\bar{H}(5^*)$. Σ acquires the vacuum expectation value (vev) $\langle \Sigma \rangle = \sigma \text{diag}(2, 2, 2, -3, -3)$, where $\sigma \simeq M_{\text{GUT}}$, so that SU(5) is broken to G_{SM} . The pair of quintets then breaks G_{SM} to SU(3) \times U(1)_{em} at M_{ew} ; it contains the SM Higgs doublets, H_f and \bar{H}_f , which break G_{SM} , and color triplets, H_C and \bar{H}_C , respectively. Including possible terms up to order $1/M_{\text{P}}$, the Yukawa couplings read

$$W = \frac{1}{4} \epsilon_{abcde} \left(Y_1^{ij} 10_i^{ab} 10_j^{cd} H^e + f_1^{ij} 10_i^{ab} 10_j^{cd} \frac{\Sigma_f^e}{M_{\text{P}}} H^f + f_2^{ij} 10_i^{ab} 10_j^{cd} H^d \frac{\Sigma_f^e}{M_{\text{P}}} \right) + \sqrt{2} \left(Y_2^{ij} \bar{H}_a 10_i^{ab} 5_{jb}^* + h_1^{ij} \bar{H}_a \frac{\Sigma_b^a}{M_{\text{P}}} 10_i^{bc} 5_{jc}^* + h_2^{ij} \bar{H}_a 10_i^{ab} \frac{\Sigma_b^c}{M_{\text{P}}} 5_{jc}^* \right), \quad (1)$$

thus

$$Y_u = Y_1 + 3 \frac{\sigma}{M_{\text{P}}} f_1^S + \frac{1}{4} \frac{\sigma}{M_{\text{P}}} (3f_2^S + 5f_2^A), \quad (2)$$

$$Y_d = Y_2 - 3 \frac{\sigma}{M_{\text{P}}} h_1 + 2 \frac{\sigma}{M_{\text{P}}} h_2,$$

$$Y_e = Y_2 - 3 \frac{\sigma}{M_{\text{P}}} h_1 - 3 \frac{\sigma}{M_{\text{P}}} h_2.$$

Here $\sigma/M_{\text{P}} \sim \mathcal{O}(10^{-2})$, and S and A denote the symmetric and antisymmetric parts of the matrices. From Eqs. (2) one reads off,

$$Y_d - Y_e = 5 \frac{\sigma}{M_{\text{P}}} h_2, \quad (3)$$

hence the failure of Yukawa unification is naturally accounted for by the presence of h_2 .

The dimension-five operators that lead to proton decay arise from the couplings of quarks and leptons to H_C and \bar{H}_C , which acquire masses $\mathcal{O}(M_{\text{GUT}})$. If we integrate them out, two operators remain [13],

$$W_5 = \frac{1}{M_{H_C}} \left[\frac{1}{2} Y_{qq} Y_{ql} Q Q Q L + Y_{ue} Y_{ud} u^c e^c u^c d^c \right], \quad (4)$$

where

$$\begin{aligned}
Y_{qq} &= Y_{qq}^S = Y_{ue}^S = Y_u^S - 5 \frac{\sigma}{M_P} \left(f_1^S + \frac{1}{4} f_2^S \right), \\
Y_{ue}^A &= Y_u^A - \frac{5}{2} \frac{\sigma}{M_P} f_2^A, \\
Y_{ql} &= Y_e + 5 \frac{\sigma}{M_P} h_1, \\
Y_{ud} &= Y_d + 5 \frac{\sigma}{M_P} h_1.
\end{aligned} \tag{5}$$

The entries in f_j and h_j can lead to a simple pattern of these Wilson coefficients with small entries only [5].

In $\text{SO}(10)$, the analogous five-dimensional operator is $16 \ 16 \ 10_H \ 45_H$. Here, we use the scenario, where $\text{SO}(10)$ is broken to $\text{SU}(5)$ by a pair $16_H \oplus 16_H^*$ (cf. the discussion at the end of this letter). $\text{SU}(5)$ is broken to G_{SM} by the adjoint 45_H which includes the $\Sigma(24)$ of $\text{SU}(5)$; finally the breaking of G_{SM} is achieved by 10_H which includes both $H(5)$ and $\bar{H}(5^*)$ of $\text{SU}(5)$. This breaking pattern guarantees that the $\text{SO}(10)$ gauge coupling constant remains perturbative up to M_P [11].

In order express the $\text{SO}(10)$ in $\text{SU}(5)$ fields, we consider a set of operators b_j ($j = 1, \dots, 5$) plus their Hermitean conjugates, b_j^\dagger , satisfying [14]

$$\{b_i, b_j\} = \{b_i^\dagger, b_j^\dagger\} = 0, \quad \{b_i, b_j^\dagger\} = \delta_{ij}. \tag{6}$$

With Γ matrices, defined as

$$\Gamma_{2j-1} = -i (b_j - b_j^\dagger), \quad \Gamma_{2j} = (b_j + b_j^\dagger), \tag{7}$$

we can construct the generators of $\text{SO}(10)$, $\Sigma_{\mu\nu}$, as

$$\Sigma_{\mu\nu} = \frac{1}{2i} [\Gamma_\mu, \Gamma_\nu]. \tag{8}$$

The spinor representation can be split into two 16-dimensional representations Ψ_\pm by chiral projection, $\frac{1}{2}(1 \pm \Gamma_0)$, $\Gamma_0 = i \prod_j \Gamma_j$. We define an $\text{SU}(5)$ invariant vacuum state $|0\rangle$ and expand the spinors in terms of $\text{SU}(5)$ fields. The SM fermions are assigned to 16,

$$\begin{aligned}
16 = |\Psi_+\rangle &= |0\rangle \psi_0 + \frac{1}{2!} b_i^\dagger b_j^\dagger |0\rangle \psi^{ij} \\
&+ \frac{1}{4!} \epsilon^{ijklm} b_j^\dagger b_k^\dagger b_l^\dagger b_m^\dagger |0\rangle \bar{\psi}_i,
\end{aligned} \tag{9}$$

where we identify $\bar{\psi}_i$ and ψ^{ij} with 5^* and 10 of $\text{SU}(5)$, respectively. The singlet ψ_0 denotes the left-handed anti-neutrino.

The fundamental representation can be written in $\text{SU}(5)$ fields as

$$\phi_\mu = \begin{cases} \phi_{2j} &= \frac{1}{2} (\phi_{c_j} + \phi_{\bar{c}_j}) \\ \phi_{2j-1} &= \frac{1}{2i} (\phi_{c_j} - \phi_{\bar{c}_j}) \end{cases}, \tag{10}$$

where ϕ_{c_j} and $\phi_{\bar{c}_j}$ transform like $\text{SU}(5)$ representations. Thus we are able to compute the $\text{SO}(10)$ in $\text{SU}(5)$ fields which then only have to be reduced to irreducible representations. We obtain

$$\Gamma_\mu \phi_\mu = b_j \phi_{c_j} + b_j^\dagger \phi_{\bar{c}_j}. \tag{11}$$

To have a canonical kinetic term for the $\text{SU}(5)$ Higgs fields, we normalize the fields by

$$\phi_{\bar{c}_j} = \sqrt{2} \ 5_{Hj}, \quad \phi_{c_j} = \sqrt{2} \ 5_{Hj}^*. \tag{12}$$

Now we are able to express the basic Yukawa couplings,

$$16 \ 16 \ 10_H = \langle \Psi_+^* | B \Gamma_\mu | \Psi_+ \rangle \phi_\mu, \tag{13}$$

in $\text{SU}(5)$ fields. The matrix B is the equivalent of the charge conjugation matrix \mathcal{C} (which is dropped here) for $\text{SO}(10)$. We find

$$\begin{aligned}
W_Y^{10} &= \sqrt{2} i f_{ab} [-(1_a \ 5_b^* + 5_a^* \ 1_b) 5_H \\
&+ (10_a \ 5_b^* + 5_a^* \ 10_b) 5_H^* + \frac{1}{4} 10_a \ 10_b \ 5_H].
\end{aligned} \tag{14}$$

Additionally to the known $\text{SU}(5)$ couplings (which are symmetric now), we observe the couplings for the neutrinos, leading to their Dirac masses m_ν^D , with $m_\nu^D = m_u$.

Next, let us turn to $16 \ 16 \ 10_H \ 45_H$. It appears in four different invariants,

$$(16 \ 16)_{10} (10_H \ 45_H)_{10}, \tag{15a}$$

$$(16 \ 16)_{120} (10_H \ 45_H)_{120}, \tag{15b}$$

$$(16 \ 10_H)_{16^*} (16 \ 45_H)_{16}, \tag{15c}$$

$$(16 \ 10_H)_{144^*} (16 \ 45_H)_{144}. \tag{15d}$$

Note that in Ref. [11], only the second term (15b) is studied. To calculate the different couplings, we generalize Eqn. (10) so that [12]

$$\begin{aligned}
45 : \quad \Sigma_{\mu\nu} \phi_{\mu\nu} &= -i \left(b_i^\dagger b_j^\dagger \phi_{c_i c_j} + b_i b_j \phi_{\bar{c}_i \bar{c}_j} \right. \\
&\quad \left. + 2 b_i^\dagger b_j \phi_{c_i \bar{c}_j} - \phi_{c_n \bar{c}_n} \right),
\end{aligned} \tag{16}$$

$$\begin{aligned}
120 : \quad \Gamma_\mu \Gamma_\nu \Gamma_\lambda \phi_{\mu\nu\lambda} &= b_i^\dagger b_j^\dagger b_k^\dagger \phi_{c_i c_j c_k} + b_i b_j b_k \phi_{\bar{c}_i \bar{c}_j \bar{c}_k} \\
&+ 3 b_i^\dagger b_j b_k \phi_{c_i \bar{c}_j \bar{c}_k} + 3 b_i^\dagger b_j^\dagger b_k \phi_{c_i c_j \bar{c}_k} \\
&+ b_i \phi_{\bar{c}_n c_n \bar{c}_i} + b_i^\dagger \phi_{\bar{c}_n c_n c_i}.
\end{aligned} \tag{17}$$

The tensors of $\phi_{\mu\nu}$ can be decomposed into their irreducible forms as

$$\begin{aligned}
\phi_{c_n \bar{c}_n} &= h, & \phi_{c_i c_j} &= h^{ij}, \\
\phi_{\bar{c}_i \bar{c}_j} &= h_{ij}, & \phi_{c_i \bar{c}_j} &= h_j^i + \frac{1}{5} \delta_j^i h.
\end{aligned} \tag{18}$$

with the 1, 10^* , 10 and 24-plet, normalized as

$$\begin{aligned}
h &= \sqrt{10} H, & h_{ij} &= \sqrt{2} H_{ij}, \\
h^{ij} &= \sqrt{2} H^{ij}, & h_j^i &= \sqrt{2} H_j^i.
\end{aligned} \tag{19}$$

Analogously, we have for $\phi_{\mu\nu\lambda}$

$$\begin{aligned} \phi_{c_i c_j \bar{c}_k} &= f_k^{ij} + \frac{1}{4} (\delta_k^i f^j - \delta_k^j f^i), \\ \phi_{c_i \bar{c}_j \bar{c}_k} &= f_{jk}^i - \frac{1}{4} (\delta_j^i f_k - \delta_k^i f_j), \\ \phi_{c_i c_j c_k} &= \epsilon^{ijklm} f_{lm}, \quad \phi_{\bar{c}_i \bar{c}_j \bar{c}_k} = \epsilon_{ijklm} f^{lm}, \\ \phi_{\bar{c}_n c_n c_i} &= f^i, \quad \phi_{\bar{c}_n c_n \bar{c}_i} = f_i. \end{aligned} \quad (20)$$

We identify the 5, 10, 45, 5*, 10* and 45*-plet of 120, which are normalized as

$$\begin{aligned} f^i &= \frac{4}{\sqrt{3}} h^i, \quad f^{ij} = \frac{1}{\sqrt{3}} h^{ij}, \quad f_k^{ij} = \frac{2}{\sqrt{3}} h_k^{ij}, \\ f_i &= \frac{4}{\sqrt{3}} h_i, \quad f_{ij} = \frac{1}{\sqrt{3}} h_{ij}, \quad f_{jk}^i = \frac{2}{\sqrt{3}} h_{jk}^i. \end{aligned} \quad (21)$$

For the first invariant (15a) we need the coupling 10 – 10 – 45, which can be decomposed as

$$\begin{aligned} \sqrt{2} [(5_{10} 5_{10}^* + 5_{10}^* 5_{10}) 1_{45} + 5_{10} 5_{10} 1_{45}^* \\ + 5_{10}^* 5_{10}^* 1_{45} + (5_{10} 5_{10}^* + 5_{10}^* 5_{10}) 24_{45}]. \end{aligned} \quad (22)$$

Since the vev of the 45_H is taken in the 24-direction of SU(5), only the last two terms are relevant. Now we integrate out the heavy field 10 in Eqs. (14, 22) by means of $W_M^{10} = 2 M_{10} 5 5^*$ and obtain the coupling given in Eqn. (29a).

The calculation for the second term (15b) is straightforward. We compute

$$\begin{aligned} W_Y^{120} &= \frac{i}{\sqrt{3}} f_{ab} [(-1_a 10_b + 10_a 1_b) 10_H^* + 2 \cdot 5_a^* 5_b^* 10_H \\ &\quad + 2 (1_a 5_b^* - 5_a^* 1_b) 5_H + (5_a^* 10_b - 10_a^* 5_b^*) 5_H^* \\ &\quad - \frac{1}{2} 10_a 10_b 45_H + (5_a^* 10_b - 10_a^* 5_b^*) 45_H^*] \end{aligned} \quad (23)$$

and calculate the relevant terms of the coupling 10 – 45 – 120,

$$\begin{aligned} \sqrt{3} [2 (5_{10} 24_{45} 45_{120}^* + 5_{10}^* 24_{45} 45_{120}) \\ - 5_{10} 24_{45} 5_{120}^* - 5_{10}^* 24_{45} 5_{120}] + \dots \end{aligned} \quad (24)$$

With the mass term

$$W_M^{120} = M_{120} (\frac{1}{2} 10 10^* + 45 45^* - 2 \cdot 5 5^*), \quad (25)$$

we then get the result of Eqn. (29b).

The remaining two operators read

$$(16 10_H)_{16^*} (16 45_H)_{16} = \tilde{\Psi} B \Gamma_\mu \phi_\mu \Sigma_{\nu\rho} \Psi \phi_{\nu\rho}, \quad (26)$$

$$\begin{aligned} (16 10_H)_{144^*} (16 45_H)_{144} &= \tilde{\Psi} B \phi_\mu \Gamma_\nu \Psi \phi_{\mu\nu} \\ &\quad - (\text{Eqn. (26)}). \end{aligned} \quad (27)$$

The first expression in Eqn. (27) describes the reducible 160 representation. Since the 144 requires

$$\Gamma_\mu \tilde{\phi}_\mu = 0, \quad (28)$$

we add $\Gamma_\mu^2 = 1$ to project out the 16 contribution which is already calculated in Eqn. (26). Then we get the 144 contribution just by the difference of the two terms.

Altogether, the couplings of the four operators read

$$\begin{aligned} \hat{Y}_{10} &= \frac{h_{ij}^{10}}{M_{10}} \left\{ \frac{1}{2} \epsilon_{abcde} 10_i^{ab} 10_j^{cd} \Sigma_f^e H^f \right. \\ &\quad \left. - 2 \bar{H}_a \Sigma_b^a (10_i^{bc} 5_{jc}^* + 10_j^{bc} 5_{ic}^*) \right\} \end{aligned} \quad (29a)$$

$$\begin{aligned} \hat{Y}_{120} &= \frac{h_{ij}^{120}}{M_{120}} \left\{ -2 \epsilon_{abcde} 10_i^{ab} 10_j^{cd} H^d \Sigma_f^e \right. \\ &\quad - \bar{H}_a \Sigma_b^a (10_i^{bc} 5_{jc}^* - 10_j^{bc} 5_{ic}^*) \\ &\quad \left. - 4 \bar{H}_a \Sigma_b^c (10_i^{ab} 5_{jc}^* - 10_j^{ab} 5_{ic}^*) \right\} \end{aligned} \quad (29b)$$

$$\begin{aligned} \hat{Y}_{16} &= \frac{h_{ij}^{16}}{M_{16}} \left\{ \frac{1}{2} \epsilon_{abcde} 10_i^{ab} 10_j^{cd} H^d \Sigma_f^e \right. \\ &\quad \left. + 2 \bar{H}_a \Sigma_b^a 10_i^{bc} 5_{jc}^* - \bar{H}_a 10_i^{ab} \Sigma_b^c 5_{jc}^* \right\} \end{aligned} \quad (29c)$$

$$\begin{aligned} \hat{Y}_{144} &= \frac{h_{ij}^{144}}{M_{144}} \left\{ \epsilon_{abcde} 10_i^{ab} 10_j^{cd} \Sigma_f^e H^f \right. \\ &\quad - \frac{1}{2} \epsilon_{abcde} 10_i^{ab} 10_j^{cd} H^d \Sigma_f^e \\ &\quad \left. + 2 \bar{H}_a \Sigma_b^a 10_i^{bc} 5_{jc}^* + \bar{H}_a 10_i^{ab} \Sigma_b^c 5_{jc}^* \right\}, \end{aligned} \quad (29d)$$

where we only list the SU(5) relevant terms. Without loss of generality, we can assume that the heavy particles all have the same mass and can compare the couplings with those of SU(5) (cf. Eqn. (1)),

$$f_1 = \frac{1}{2} h^{10} + h^{144}, \quad (30a)$$

$$f_2 = -2 h^{120} + \frac{1}{2} h^{16} - \frac{1}{2} h^{144}, \quad (30b)$$

$$h_1 = -2 h^{10} - h^{120} + 2 h^{16} + 2 h^{144}, \quad (30c)$$

$$h_2 = -4 h^{120} - h^{16} + h^{144}. \quad (30d)$$

Here, h^{10} is symmetric whereas h^{120} is antisymmetric; h^{16} and h^{144} , are not constrained by symmetry requirements. We see that SO(10) does not restrict the contributions from the higher-dimensional operators, contrary to the basic Yukawa couplings (cf. Eqn. (13)). With these equations, we reduce the SO(10) case down to SU(5), so the implications of the higher-dimensional operators for proton decay in SO(10) are the same as in SU(5) [5].

If we consider the complete breaking of SO(10) to G_{SM} , more five-dimensional operators can appear. SO(10) can be broken via a pair $16_H \oplus 16_H^*$, where the SU(5) singlet component obtains a vev $\mathcal{O}(M_{\text{GUT}})$. Then the two new dimension-five operators, $16 16 16_H 16_H$ and $16 16 16_H^* 16_H^*$, generate Majorana masses for the right-handed neutrinos. If, moreover, the 5_{16}^* and 5_{16} acquire vevs as well (as in Refs. [9]), these operators allow additional contributions to Eqn. (30). The second coupling

was partially worked out in Refs. [12]. Alternatively, if we use a 210_H to break $SO(10)$, additional terms can arise, since the 210 includes a 24 of $SU(5)$ [15].

We extended the analysis of higher-dimensional operators in $SU(5)$ to $SO(10)$. In contrast to the basic Yukawa couplings, these operators are not restricted compared to $SU(5)$. In the simple case, where $SO(10)$ is broken to G_{SM} via 16_H , 16_H^* and 45_H and the former have only vevs in the $SU(5)$ singlet direction, these represent all possible operators of dimension five. Hence, it would be interest-

ing to study if this model, with only the Yukawa couplings $16\ 16\ 10_H$ and the dimension-five operators studied in this paper (including those which generate Majorana masses for the right-handed neutrinos), both reproduces the fermion masses and mixing angles and is consistent with the experimental limit on proton decay.

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